

Noise Assessment Framework for Optimizing ECG Key Generation

Nima Karimian, Fatemeh Tehranipoor, Zimu Guo*, Mark Tehranipoor* and Domenic Forte*

University of Connecticut, Storrs, CT, 06269, USA

*University of Florida, Gainesville, FL, 32611, USA

Abstract—Bioelectrical signals such as electrocardiogram (ECG) have shown promise as biometrics, but their continuous nature and drastic acquisition variations make it difficult to deploy them for biometric-based key generation. In particular, it is nearly impossible to obtain raw ECG measurements from a large population under all possible test conditions. In this paper, we build upon our recent approach called *IOMBA* by combining it with a pre-assessment framework that uses synthetic ECGs to characterize the impact of different sources of noise on ECG-based keys. Our framework uses an auto-regressive (AR) model with three modulated sources of noise - baseline wander (BW), electromyography (EMG), and motion artifact (MA). The performance of the proposed framework is validated using normal ECG signals from popular ECG databases. Different feature extraction methods are applied for ECG key generation and the performance of each approach with each noise source is evaluated. The proposed framework can be used to optimize pre-processing approaches for low-cost applications.

Index terms— Biometric, ECG, NA-IOMBA, Key generation, Synthetic noise, Synthetic ECG, Biometric Quantization.

Electrocardiogram (ECG) and Photoplethysmography (PPG) are a internal physiological human signal that can be used for biometric authentication [1]–[3] with measuring simple non-invasive and low-cost sensors. An electrocardiogram (ECG) is a measure of electrical activity of the heart which is primarily used to diagnose cardiovascular disease. Recently, it has been shown that the ECG from person-to-person is unique and may be distinctive enough for biometric applications [1], [4], [5]. A major challenge in using ECG in biometric applications is that the signal is often contaminated by noise and artifacts such as power line interface, baseline drift, motion artifacts, arrhythmias, and electromyography (EMG). Such noise may lie within the frequency band of interest and can manifest with similar morphology as ECG making it difficult to remove [6]. While approaches based on fuzzy extraction and error correction code (ECC) [7] have shown promise for generation of keys from noisy biometric signals, these may not be suitable for ECG. For example, in body area network (BAN) and implantable medical device (IMD) applications, the pre-processing, feature extraction, key generation, error correction, etc. must be performed by very simple devices with little power, storage, and hardware resources.

In this paper, we aim to overcome such reliability issues by

directly assessing the impact of all major sources of noise on ECG key generation. Since each of the noise sources have different characteristics in terms of frequency band and amplitude, their impacts on key generation and the cost to mitigate them with pre-processing and fuzzy extraction shall be different. Through a quantitative assessment, it may be possible to optimize ECG key generation on a user-to-user basis for resource constrained systems. To the best of our knowledge, there has not been any prior research specifically investigating the impact of noise on ECG key generation. Furthermore, the basis of our approach are synthetic ECGs which enables us to avoid exhaustive measurement of ECGs for the assessment.

Our main contributions are as follows:

- 1. Synthetic ECG signal:** In order to perform this research, our biggest challenge lies in how to obtain noisy ECG data. Put simply, it is impossible to take raw ECG measurements from a large population under all possible test conditions. Thus, in this paper we discuss how dynamical models can be used to generate synthetic ECG signals.
- 2. ECG Noise Modeling:** We investigate the major noise sources and apply non-stationary real ECG noise source with different signal-to-noise-ratios (SNR) to the synthetic ECGs.
- 3. Feature Extraction:** We review low-cost, non-fiducial point ECG feature extraction algorithms (Normalize- Convolved Normalize (NCN), normalized autocorrelation (AC), and morphological of ECG signal) and investigate the impact of noise sources on each.
- 4. Noise Aware IOMBA (NA-IOMBA) scheme:** We applied re-optimization *IOMBA* margins module to determine new margins based on feedback from the assessment (i.e., smallest margins to the most reliable features; highest margins to least reliable features). *NA-IOMBA* scheme can improve the reliability of biometric key without pre-processing (filtering) requirement that can be useful for low-cost/power application such as IoT devices.

In the following sections, we first present the background of electrocardiogram (ECG) including ECG synthetic model, and ECG noise modeling in section II. In Section III, we present the filtering and feature extraction techniques. The *IOMBA* based biometric key generation are described in section IV. Assessment framework for noise aware *IOMBA*, experiments and results are discussed in Section V. In Section VI, conclusions and plans for future work are provided.

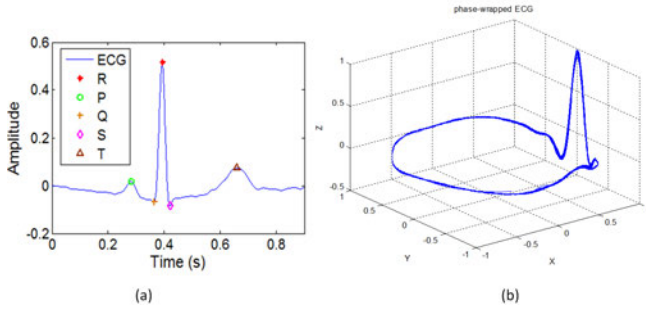


Figure 1. Typical normal ECG signal. (a) One beat normal ECG signal with fiducial point, (b) trajectories several cycles of the ECG phase-wrapped in the Cartesian coordinates.

I. ELECTROCARDIOGRAM (ECG) PRELIMINARIES

The ECG signal is generated by electrical current of the heart (see Fig. 1). The shapes of the ECG waveform depend on the anatomic features of the human body and heart, and thus are distinctive from one person to another.

A. ECG synthetic model

Since the ECG is constructed from multiple peaks such as P, Q, R, S and T wave which are like a Gaussian functions with different amplitudes and widths, McSharry et al. proposed a non-linear dynamical model for generating realistic synthetic ECG signals using three ordinary differential equations. The model consists of a circular limit cycle of unit radius in the (x, y) plane around which the trajectory is pushed up and down as it approaches the P, Q, R, S and T points in the ECG. It is given by the following equations [8]

$$\begin{cases} \dot{x} = \alpha x - \omega y \\ \dot{y} = \alpha y + \omega x \\ \dot{z} = -\sum_{i \in P, Q, R, S, T} \alpha_i \Delta \theta_i \exp\left[-\frac{\Delta \theta_i^2}{2b_i^2}\right] - (z - z_0) \end{cases} \quad (1)$$

where $\alpha = 1 - \sqrt{x^2 + y^2}$, $\Delta \theta_i = (\theta - \theta_i) \bmod 2\pi$, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$, the angular position of the elements of x, y , range over $[-\pi, \pi]$, and ω is the angular velocity of the trajectory as it moves around the limit cycle. z_0 is the contribution from the baseline that is assumed to be a relatively low frequency signal component coupled with the respiratory sinus frequency (RSA). Three dimensional (3-D) trajectory in a 3-D state space with coordinates (x, y, z) and its phase-wrapped trace in the 3-D model space are shown in Figure 1(b). In this literature we have normalized the amplitude of a single ECG cycle for simplicity. It can be seen that each component of the ECG signal is modeled with a Gaussian kernel which has three parameters α_i , b_i and θ_i by neglecting the baseline term $(z - z_0)$ in Eq. equation (1). The times and angles are relative to the position of the R peak since it is always assumed to have zero phase and the ECG contents lying between two consecutive R peaks are assumed to have a phase between $[-\pi, \pi]$ (see Figure 1). Thus, the phase signal θ is available by simply detecting the R peaks. To estimate the dynamic model parameters for the given ECG, mean and variance of

the phase-wrapped ECG is calculated for all phases between $-\pi$ and π which are depicted in Figure 1.

The dynamic state equations proposed by McSharry et al. can also be transformed into polar coordinates as follows [9]

$$\begin{cases} \dot{r} = r(1 - r) \\ \dot{\theta} = \omega \\ \dot{z} = -\sum_{i \in P, Q, R, S, T} \alpha_i \Delta \theta_i \exp\left[-\frac{\Delta \theta_i^2}{2b_i^2}\right] - (z - z_0) \end{cases} \quad (2)$$

The first equation in (Eq. equation (2)) shows the circular behavior of the generated trajectory by the model. second and third equations in (Eq. equation (2)) are independent from r , making the first equation redundant. Therefore the first equation may be excluded as it has no effect on the synthetic ECG.

B. ECG Noise Modeling

Raw ECG signals contain both high and low frequency noise components which are often non-stationary in time. Baseline wander (BW) is an extraneous and low frequency activity in the ECG signal. EMG noise is caused by the electrical activity of skeletal muscles during periods of contraction. Motion artifacts (MA) are transient baseline changes caused by changes in the electrode skin impedance with electrode motion. Time-varying auto-regressive (AR) parametric models can be applied to generate realistic ECG noise which follow the non-stationary characteristics and the spectral shape of real noise. Thus, it is possible to generate noise with different variances through a time-varying AR model. The parameters of this model are trained by using real noises such as NSTDB [10]. To estimate the time-varying AR parameters, a standard Kalman Filter (KF) is used [11]. For the time series of y_n , a time-varying AR model of order p can be written as follows:

$$y_n = -\sum_i^p a_n(i) y_{n-i} + v_n \quad (3)$$

where v_n is the input white noise and $a_n(i)$ ($i = 1, \dots, p$) coefficients are the p time-varying AR parameters at the time instance of n . By defining $x_n = [a_n(1), a_n(2), \dots, a_n(p)]^T$ as a state vector, and $h_n = [y_{n-1}, y_{n-2}, \dots, y_{n-p}]^T$ as observation model, we can reformulate the problem of AR parameter estimation in the KF form. We refer the reader to [11] for more details. Having the time-varying AR model, we later generate synthetic BW, EM, and MA noise by the propose method with different signal-to-noise ratio (SNR). Since the sampling rate of original source noises and ECG signals are 360 and 1000 Hz, the synthetic noises are re-sampled to 1000 Hz.

II. PRE-PROCESSING

A. Filtering & R peak det.

In this paper, we have employed a 4th Butterworth band pass filter with cutoff frequency 1Hz-40Hz to eliminate various kinds of noise in ECG signals. We use the Pan-Tompkins [12] for R peak detection. Then, we consider a fixed window

by taking an identified R peak as a reference to segment ECG signal in terms of RR intervals. Note that the filtering process is only considered for generating a reference (i.e., noise reduction) ECG key. A noisy signal is still used to generate keys in many of our results.

B. Feature Extraction Approaches

Feature extraction is the process by which important features of a sample are selected or enhanced. For biometric-based identification and key generation, features selected should be distinctive, independent, and reliable. In this paper, we analyze noise for the following feature selection approaches that have been applied on ECGs.

(1) Morphology of ECG: Since each heartbeat contains a series of important components such as P, Q, R, S and T peaks (Fig. 2), one can consider the window over the R peak in each beat to reach the segmented ECG. Note that the segmented ECG signal (entire heartbeat) is called morphology of the ECG signal. The choice of using the entire heartbeat morphology have been used with the intention of keeping the computational complexity low, using only R peak detection. Another advantage of using entire heartbeat morphology is it can avoid the detection of the fiducial points related with other waveform components (e.g., P wave and T wave), which are usually more sensitive to noise due to their relatively lower magnitudes.

(2) Normalized Autocorrelation (AC): The ECG signal is a non-stationary signal and consists of repetitive waveform patterns. The motivation behind this non-fiducial approach is the use of normalized autocorrelation (AC) method on non-overlapping windows of the filtered individual ECG signal without the use of fiducial point(landmark) detection [13]. In order to extract the feature vector representing the ECG's signature, a windowed ECG signal and estimation of the normalized AC over a window of m are taken into account. In fact, autocorrelation gives an automatic shift invariant feature set that represents repetitive characteristics over multiple heartbeat cycles. The autocorrelation coefficients can be written as:

$$R[m] = \frac{1}{R[0]} \sum_i s[i]s[i+m] \quad (4)$$

where $s[i]$ is the ECG signal at time i and m is chosen greater than the mean QRS duration.

(3) Normalize-Convolved Normalize (NCN): Normalize-Convolved Normalize (NCN) is used in [14]. The approach focuses on the QRS complex which is more invariant than other peaks over time. It begins by detecting the R peak. Then an equal number of sample points from both sides of the identified R wave are selected.

III. IOMBA BASED BIOMETRIC KEY GENERATION

The noise that distorts a user's ECG signal cannot be completely removed even by state-of-art pre-processing techniques. While such noise can introduce false positives and false negatives for authentication, it presents an even greater challenge for key generation where not even a single error

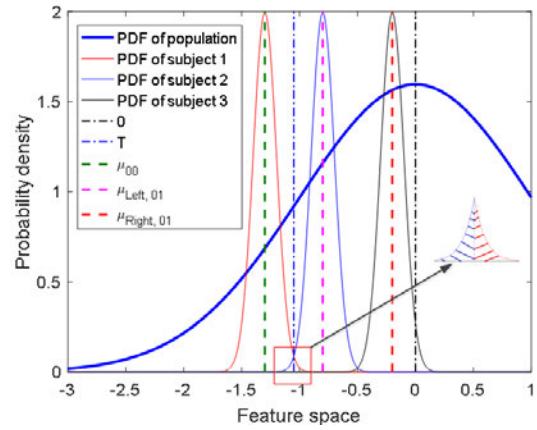


Figure 2. Schematic for optimization of quantization with two bits.

can be tolerated. In our prior work, we have developed a statistical framework called interval optimized mapping bit allocation (*IOMBA*), which takes a noisy ECG signal as input and generates reliable binary keys along with the necessary helper data [1]. In the prior work we have shown that it is possible to generate 217 key bits from ECG with 99.9% reliability and high entropy.

In short, *IOMBA* tunes the biometric key generation process to each user rather than relying on a generic approach for all users. The performance is controlled by two parameters, α and β , which define the entropy and reliability of generated keys respectively. *IOMBA* eliminates features from the space that have low entropy and quantizes features with larger entropy and lesser noise into more bits. Then, since noise will impact each user's ECG differently, the features that are more sensitive to noise (i.e., unreliable for key generation) are also removed on a user-to-user basis. As a result, the length of keys generated varies based on the α and β parameters as well as from user to user.

Initially, the ECG signals from a large population of subjects are recorded. The major steps of the *IOMBA* are then as follows.

1) Data Pre-processing: The signals from the population are pre-processed to remove noise and segmented. Then, a feature extraction approach is applied on these segments. Each of these segments produces a feature vector containing the same number of feature elements. The feature elements from the same location are extracted from the population and normalized into a standard normal distribution. Note that if a feature is determined as non-Gaussian, it is removed from consideration. The same normalization parameters are later exploited to normalize the corresponding feature elements of each subject. The normalized probability density functions (*PDF*) of the population and subjects are illustrated in Fig. 2(a) and used as a running example.

2) IOMBA Margin Calculation from Population Statistics: *IOMBA* quantizes each feature into a different number of bits. 2 bit quantization is illustrated in Fig. 2. The population *PDF* of a feature is shown in blue. As example to

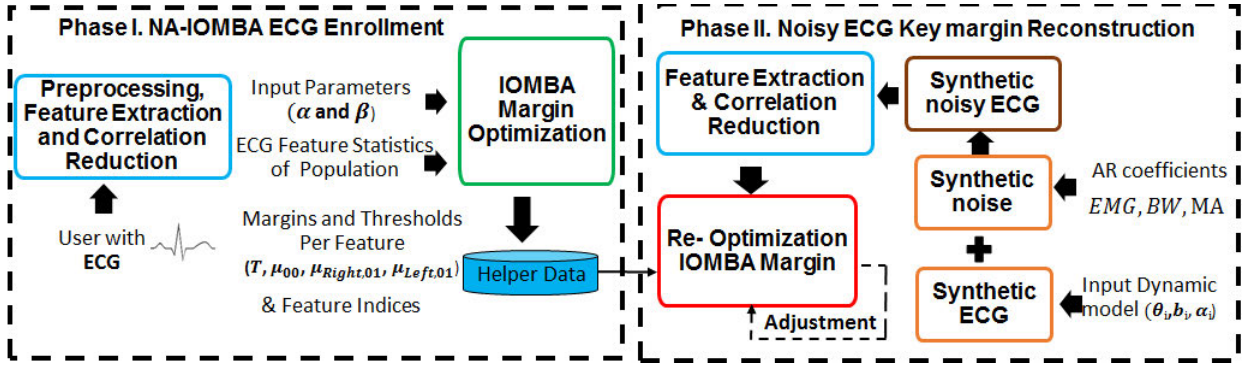


Figure 3. Block diagrams show for $NA - IOMBA$ calculation, ECG enrollment and noisy ECG key margin reconstruction phase.

illustrate the reliability calculation, three subjects distributions are considered. According to these distributions as well as the α (entropy) and β (reliability) parameters, the boundaries ($\mu_{left,01}$, $\mu_{right,01}$, μ_{00}) and threshold (0 , T) are computed by satisfying the following equations

$$\frac{\int_{\infty}^{\mu_{00}} PDF_{pop}(x)dx}{\int_{\mu_{left,01}}^{\mu_{right,01}} PDF_{pop}(x)dx} \leq \alpha \quad (5)$$

$$\begin{aligned} \int_T^{\infty} PDF_{sub_1}(x)dx &\leq \beta \\ \int_{-\infty}^T PDF_{sub_2}(x)dx &\leq \beta \\ \int_0^{\infty} PDF_{sub_3}(x)dx &\leq \beta \end{aligned} \quad (6)$$

$PDF_{pop}(x)$ denotes the distribution for feature x for the entire population and $PDF_{sub_y}(x)$ denotes the distribution (due to noise) for feature x of subject y . Fig. 2(a) illustrates these values on the negative side of the x axis. The positive boundaries and thresholds can be simply computed by mirroring the negative values onto the positive side against the y axis. These boundaries and thresholds guarantee that enrolled and regenerated key bits are statistically random and reliable. The number of boundaries (i.e., number of bits to quantize a feature into) depends on the entropy and noise expected in a feature

3) Enrollment for Key and Helper Data Generation:

For each subject, the key generation framework utilizes these boundaries to determine whether each feature element is good for generating key bits or not. The least reliable features of the user which do not fulfill the above constraints are discarded. The helper data for each subject consist of the following parts: (i) the number of bits each feature can be quantized into, (ii) the normalization parameters for each feature, and (iii) the index of the features used for the user.

4) Key Regeneration: The user presents his/her ECG, features are extracted, and the helper data stored on the system is used to eliminate the unreliable features and then quantize the reliable features to regenerate the key.

The above process can obtain a random and reliable key from each user, but can be improved in terms of accuracy and cost with a better understanding of ECG feature noise as described below.

IV. ASSESSMENT FRAMEWORK FOR NOISE AWARE IOMBA

As mentioned previously, it is impossible to acquire ECGs at all conditions, and therefore $IOMBA$'s margin calculation and enrollment may not account for all levels and types of noise experienced by ECGs in practice. Furthermore, there may be resource constrained scenarios where all pre-processing steps are too costly or energy consuming to perform. To accommodate these issues, we propose this assessment framework that can predict the impact of different noise sources and their scale on ECG key generation. Our approach exploits the prior synthetic ECG modeling and noise models from Sections II.

In short, a synthetic ECG signal is generated from a pre-processed ECG and synthetic noise is added to predict its impact on key generation. Depending on the estimated impact and expected noise, $IOMBA$ margins and boundaries are recomputed. While the original $IOMBA$ personalizes features for each user, the proposed assessment framework builds on this personalization approach by accounting for expected noise. This cannot only improve the reliability under different conditions but also allow for tuning the hardware resources needed to remove noise on a similar per user basis. For low-cost/power applications, significant resources could be saved as a result.

A. $NA - IOMBA$

Fig. 3 presents the experimental protocol of $NA - IOMBA$ scheme. In $NA - IOMBA$ ECG enrollment phase, each user ECG signal is pre-processed by applying filtering, feature extraction, and correlation reduction. Then in $IOMBA$ margins optimization process, margins and thresholds are calculated based on inputs α (reliability parameter), β (entropy parameter), and population statics parameters generated from each feature. Since $IOMBA$ features may differ from one person to another, the indices corresponding to the selected features (reliable features) need to be stored in helper data for later usage.

In noisy ECG key margin reconstruction (phase II), dynamic model parameters (b_i, θ_i, α_i) from original ECG signal are considered as the input of synthetic ECG module. The time-varying AR coefficient described in Section I-B are then

employed to add noise of desired SNR to the clean (synthetic) ECG. During the next step, the feature extraction method and correlation reduction are applied over the synthetic noisy ECGs. Meanwhile, information from helper data such as feature indices, boundaries, and thresholds are applied as re-optimization *IOMBA* margin calculation input to extract the same features as before. Finally, re-optimization *IOMBA* margins module determines new margins based on feedback from the assessment (i.e., smallest margins to the most reliable features; highest margins to least reliable features). Furthermore in *NA – IOMBA* the margins will shrink/increase, hence some of the user’s features that have been selected in *IOMBA* are sacrificed from *NA – IOMBA*. As a result, the average key length may decrease from original *IOMBA* margin calculation.

B. Simulation Experiments and Discussion

Databases: Publicly available databases PTB Diagnostic, which is offered from National Metrology Institute Germany [15], was used to study the performance of the proposed methods. PTB contains 52 healthy subject with 1000 Hz sample rate which are considered in this work.

Feature Selection: To evaluate our approaches for key generation, we consider all the feature selection methods previously discussed.

Quantization: After feature selection, *IOMBA* was applied to encode the real values into binary. We applied dynamic quantization with maximum number of bits per feature as three for simplicity.

Reliability: Reliability of ECG key generation represents the stability of key over different noise sources. If the all bits generated by the synthetic ECG signal of an individual are equal to the associated key (key has been produced in enrollment), it can be considered as reliable. Thus, intra-hamming distance (HD) is used to compute key reliability:

$$Reliability = 100 - \frac{1}{N} \sum_{i=1}^N (K_{Ref} \oplus K_{Syn}) \times 100\% \quad (7)$$

where K_{Ref} is the reference key (average several segments of a sample) and K_{Syn} is the key derived from the synthetic noisy ECG. Note that, N is the number of key bits.

C. Reliability Assessment

Based on *IOMBA* results, 725 bit keys with 99.21% reliability are extracted with $\beta = 0.01$ from morphological of ECG signal without noise. NCN and AC feature extraction have shorter key length (144 and 200 key bits respectively) with 98.30% and 98.91% reliability without noise. By applying *NA – IOMBA*, the key length decreases to 485, 104, and 124 from morphology, NCN, and AC respectively, and the reliability increases to 99.994%, 99.625% , and 99.991% without noise.

In order to determine the sensitivity of ECG key generation based on these feature extraction, the noisy ECG signal with different variances (SNR) has been applied as discussed in Section I-A and I-B. To view the impact of each noise source,

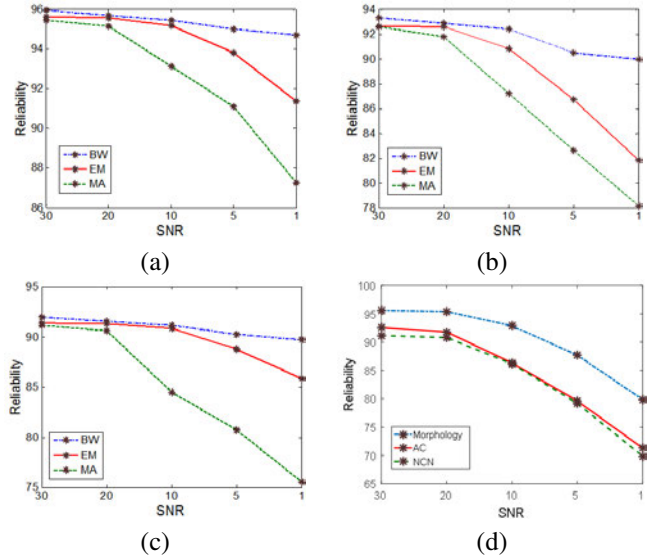


Figure 4. *IOMBA* Keys reliability rate versus input SNR; (a), (b), and (c) morphology, AC, and NCN ECG feature extractions over the separated noise sources. (d) the reliability for different feature extraction with mixed noise sources.

the synthetic ECGs were not pre-processed to remove the noise. Note that for simplicity, all the key length and reliability values shown are the average among the subjects.

Impact of ECG Noise Source on Key Reliability: Fig. 4 depicts the key reliability rate versus input SNR for ECG signal based on *IOMBA*. The SNR during the noisy segments was set to 30dB, 20dB, 10dB, 5dB, and 1dB separately. Figures 4 (a-c) indicate the impact of each noise source (BW, EM, and MA) on the reliability.

In the context of ECG noise levels, the lower SNR provides more fluctuation on ECGs (higher intra-class variation) and vice versa. Consequently there appears to be an inverse relationship between the level of generated noise and reliability. Among all these noise sources, MA and EM are the strongest noises that have a huge impact on key reliability. Our results demonstrate there is much degradation beyond 20dB except for BW noise. MA is the most troublesome noise, since it can closely mimic the element of ectopic heartbeats and the main energy of MA noises are concentrated up to 5 Hz. While the spectrum of EM noise has overlap with ECG signal and it can be extended to high frequencies. its amplitude is much lower than MA noise. The baseline noise is mainly in low frequency of 0.06 Hz (caused by breathing) which leads to a negligible impact on reliability values. Note that the maximum and minimum reliability of 95.94%, 87.17% can be achieved under BW and MA noise based on morphology feature extraction whereas they are 91.98%, 75.56%, for NCN feature extraction. Morphology feature extraction provides the best results in terms of reliability and key length. Also, the simplicity of its methodology should lead to lesser overhead (cost and power). In AC and morphology feature extraction, we have considered the entire cycle of ECG signal, while in NCN feature extraction, only the QRS complex window of

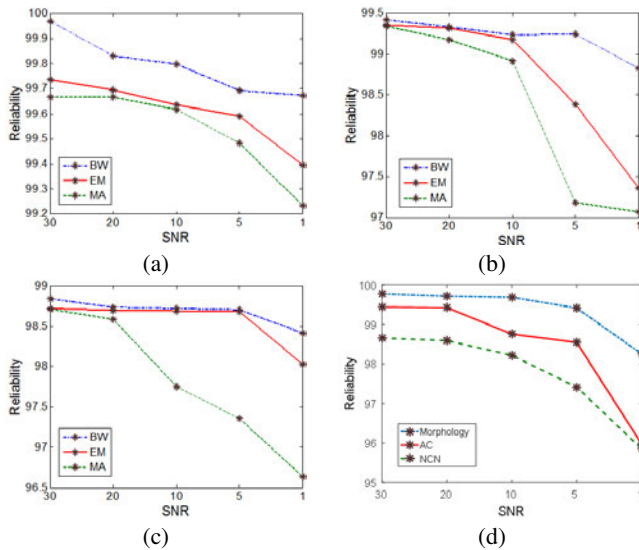


Figure 5. $NA-IOMBA$ Keys reliability rate versus input SNR; (a), (b), and (c) morphology, AC, and NCN ECG feature extractions over the separated noise sources. (d) the reliability for different feature extraction with mixed noise sources.

each cycle of ECG is used. MA noise has the most effect on QRS complex while BW and EM have the least impact on it. Therefore MA has the highest impact on NCN feature extraction, particularly at lower SNR level. On the other hand, EM noise has the least effect on QRS in terms of intra-class variation, hence the reliability of NCN is slightly higher than AC at lower SNR level. In addition, the mixed noise sources have been investigated. Based on the results, at the minimum level of SNR, the reliability for morphology ECG signal is higher than the other two feature extraction methods. The key reliability of noisy ECG signal based on $NA-IOMBA$ is shown in Fig. 5. A huge improvement in reliability is achieved. 99.96% is obtained for 30dB based on morphology. While the minimum reliability is 99.67% for 1dB. Compared to Fig. 4 (a) from $IOMBA$ technique, in $NA-IOMBA$ there is at least 4% reliability improvement. The proposed method also has an improvement over EM and MA noise in terms of reliability by 8% and 12% at 1dB noise for morphology feature extraction. Our result shows that the minimum reliability is about 96% based on mixed noise sources in 1dB, while $IOMBA$ s was just 69.88%. The huge difference (26%) indicates a significant achievement, especially considering that no pre-processing was used to remove noise.

Not that since the features with less reliability in $IOMBA$ are discarded in $NA-IOMBA$, the key length is decreased. This is an unavoidable tradeoff for this technique. Still, the key lengths are quite long especially for the morphology feature extraction approach. Since the pre-processing has not been considered in $NA-IOMBA$, the cost of reconstruction is much lower as a results which helps for saving energy consumption specific in IoT era.

V. CONCLUSION

This paper investigates the effect of noise sources on key generation from a synthetic ECG signal and how much effort is needed to remove noise on a per user basis. For low-cost/power applications (such as IoT and remote health monitoring), significant resources could be saved as a result. We propose $NA-IOMBA$ to improve the reliability of key. In addition, three feature extraction methods are applied in order to analyze their sensitivity to noise and measure the key length resulting from $IOMBA$. Having knowledge of this information, we may be able to design our hardware more efficiently in resource constrained systems. In future work, we plan on building an ECG-biometric system and applying $NA-IOMBA$ on ECGs collected in our lab to validate the proposed approaches and to estimate the cost reduction.

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